MATHEMATICS METHODS : UNITS 1 & 2, 2020

Test 3 –Trigonometric, Probability, Counting, Exponentials (10%)

(1.2.7 to 1.2.16, 1.3.1 to 1.3.17, 2.1.1 to 2.1.7)

Calculator Free - Allow 1 Minute of Reading Time

Time Allowed	First Name	Surname	Marks
20 Minutes			22 marks

Circle your Teacher's Name: Bestall Goh Fraser-Jones Freer Koulianos Luzuk Rudland Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Not Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

1. **(3 marks)**
   Expand $(x^2-2)^4$ leaving your answer in simplified form.

$1(x^2)^4(-2)^0 + 4(x^2)^3(-2)^1 + 6(x^2)^2(-2)^2 + 4(x^2)^1(-2)^3 + 1(x^2)^0(-2)^4$

$= 1x^8 - 8x^6 + 24x^4 - 32x^2 + 16$

✓ correct coeff's $(1, -8, 24, -32, 16)$
✓ correct signs
✓ correct powers of x (x^8, x^6, x^4, x^2)

2. **(4 marks)**
   Solve for x.

$$2 \times 2^{2x} - 16 = 4(2^x)$$

$2\times(2^x)^2 - 4(2^x) - 16 = 0$

Divide 2

$2^{2x} - 2\cdot 2^x - 8 = 0$

Let $y = 2^x$ ✓ must have

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$ ✓ factorises correctly

$y = 4 \quad y = -2$

$2^x = 2^2 \quad 2^x = -2$

✓ $x = 2$ No soln ✓ (indicates no soln or not valid)

or

Let $y = 2^x$ ✓ must have

$2y^2 - 4y - 16 = 0$

$(2y+4)(y-4) = 0$ ✓ factorises correctly

$2y = -4 \quad y = 4$

$y = -2$

$2^x = -2 \quad 2^x = 2^2$

No soln ✓ (indicates no soln not valid) $x = 2$ ✓

7

3. (3 marks)

Simplify the following expression.

$$\frac{3^{2n} + 9^{n+1}}{9 \times 9^{n+1}}$$

(If cancels 9^{n+1} top and bottom $\Rightarrow$ zero)

$$\frac{3^{2n} + 3^{2(n+1)}}{3^2 \times 3^{2(n+1)}}$$

$$= \frac{3^{2n} + 3^{2n} \cdot 3^2}{3^2 \times 3^{2n} \cdot 3^2}$$

✓ converts to powers of 3 and expands exp. brackets

$$= \frac{\cancel{3^{2n}}(1 + 3^2)}{\cancel{3^{2n}} \times 3^4}$$

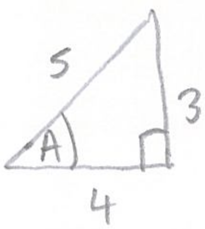
✓ factorises and cancels

$$= \frac{10}{81} \checkmark$$

4. (3, 2, 3 = 8 marks)

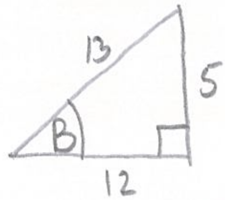
For the following trigonometric function:

a. Determine $sin(A - B)$, given that A and B are obtuse with $sin\ A = \frac{3}{5}$ and $sin\ B = \frac{5}{13}$.



$\sin A = \frac{3}{5}$

$\cos A = -\frac{4}{5}$



$\sin B = \frac{5}{13}$

$\cos B = -\frac{12}{13}$ ✓

must be negatives

$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$= \frac{3}{5} \times -\frac{12}{13} - \frac{5}{13} \times -\frac{4}{5} \checkmark$$

Subst's into rule correctly

$$= -\frac{36}{65} + \frac{20}{65}$$

$$= \frac{-16}{65} \checkmark$$

/6

b. Show that $cos\left(x+\frac{\pi}{2}\right) = -\sin(x)$

$$
\begin{aligned}
LHS &= \cos\left(x+\tfrac{\pi}{2}\right) \\
&= \cos x \cdot \cos\tfrac{\pi}{2} - \sin x \cdot \sin\tfrac{\pi}{2} \quad \checkmark \\
&= \cos x \times 0 - \sin x \times 1 \\
&= -\sin x \\
&= RHS
\end{aligned}
$$

uses rule correctly

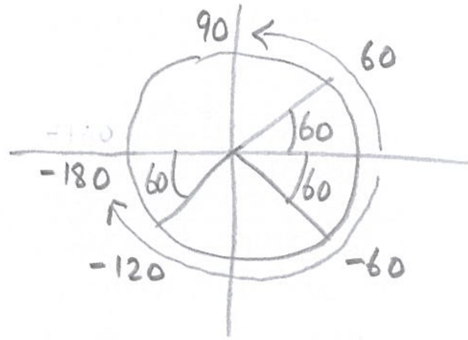
subst values and simplifies to $-\sin x$ ✓

c. Solve $4 - 4\,cos^2x = 3 \quad for\ -180° \le x \le 90°$

$$
\begin{aligned}
4 - 4\cos^2 x &= 3 \\
-4\cos^2 x &= -1 \\
\cos^2 x &= \frac{1}{4} \quad \checkmark \\
\cos x &= \pm\frac{1}{2} \quad \checkmark
\end{aligned}
$$

Simplifies

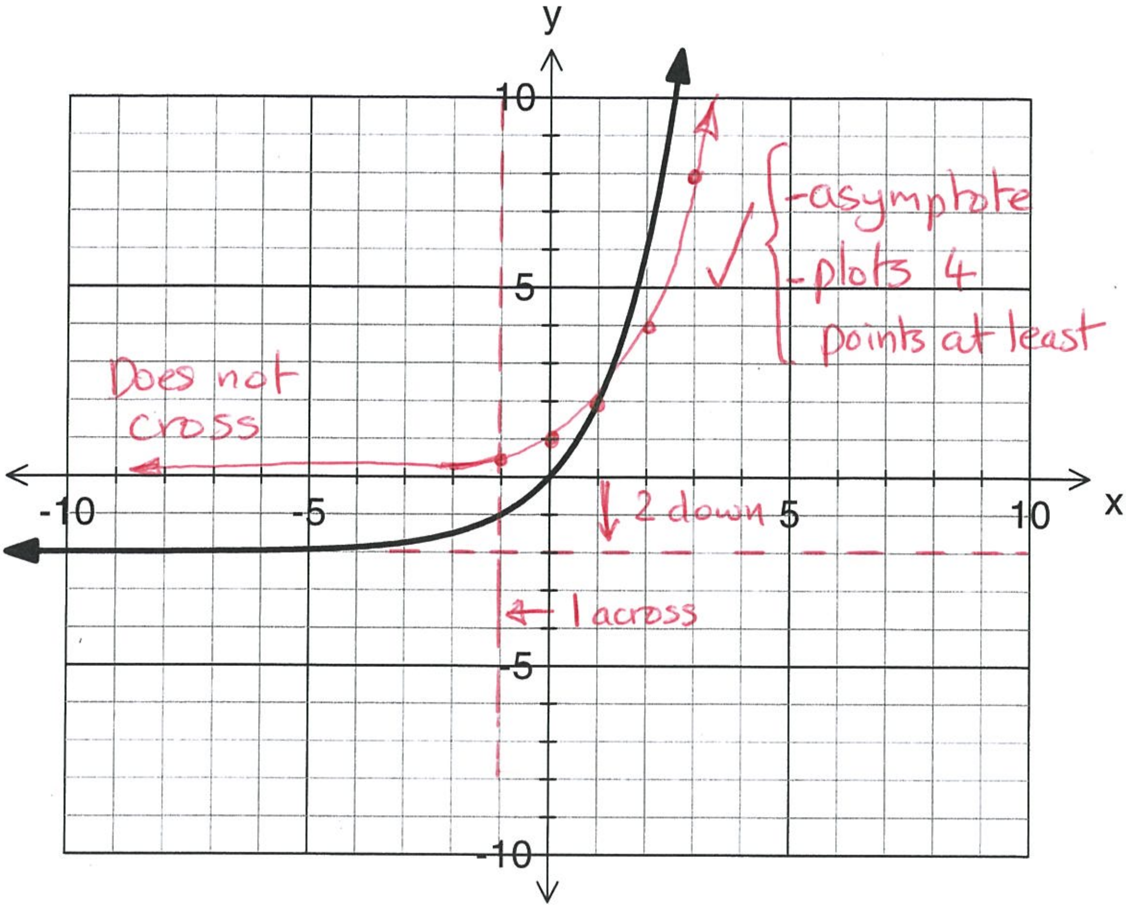
roots both sides to ge $\pm\frac{1}{2}$



$$\underbrace{x = -60, -120, 60}_{\checkmark}$$

/5

5. (2, 2 = 4 marks)



The exponential function above is a translation of $y = 2^x$.

a. Determine the equation for the function.

(must be y=)

$y = 2^{x+1} - 2$ ✓ ✓

b. Draw the graph of the function $y = 2^x$ and estimate where the two functions intersect.

Intersects at (1, 2) ✓

Second mark refer to graph

4